Mixed-Mode S-parameters in RF Design

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The evolution of semiconductor-based radios has allowed the rapid development of RF design for wireless systems and modules. One advantage of semiconductor integration is the ability to realize various radio systems as differential circuits to maximize performance and yield. However, many wireless systems are single-ended at the front-end terminals and semiconductor device design tends to be differential. Regardless if the interface is single-ended or differential, it is important to understand the characteristics of a single differential RF port in terms of Mixed-Mode S-parameters in order to bridge the two disciplines of RF design engineering and semiconductor engineering.

This article will address the most basic case of Mixed-Mode S-parameter RF design techniques: the characterization of a one-port network. Typically system designers will encounter this problem in that a semiconductor radio circuit will present a differential input port that needs to be matched and possibly converted from a differential to single-ended port. Subsequent articles will address the matching procedure and expand this analysis to multi-port systems.

The Mixed-Mode S-parameters can be measured using a single-ended Network Analyzer and S-parameter test set. Consider the basic, Mixed-mode one-port network, represented as the connection of a singled-ended two-port:

\[
[S] = \begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\]

The associated signal flow diagram for this connection of the two-port network can represented in terms of the normalized power waves for the positive and negative terminals of the single, mixed-mode port. Incident waves are \(a_P\) and \(a_N\), reflected waves are \(b_P\) and \(b_N\).
The definitions of the incident normalized power wave for the differential \((a_{DM})\) and common \((a_{CM})\) modes of the mixed mode port are defined as [1]:

\[
a_{DM} = \frac{a_p - a_N}{\sqrt{2}} \quad a_{CM} = \frac{a_p + a_N}{\sqrt{2}}
\]

The definitions of the reflected normalized power wave for the differential \((a_{DM})\) and common \((a_{CM})\) modes of the mixed mode port are defined as [1]:

\[
b_{DM} = \frac{b_p - b_N}{\sqrt{2}} \quad b_{CM} = \frac{b_p + b_N}{\sqrt{2}}
\]

The basis for these definitions will be explored in subsequent articles, when general multiport, mixed-mode systems are considered.

The differential, common and mixed-mode reflection coefficients can be defined for the mixed-mode port, respectively:

\[
S_{DID1} = \frac{b_{DM}}{a_{DM}} \quad S_{CIC1} = \frac{b_{CM}}{a_{CM}} \quad S_{DIC1} = \frac{b_{DM}}{a_{CM}} \quad S_{CID1} = \frac{b_{CM}}{a_{DM}}
\]


Using the signal flow graph, determine the terminal reflected waves, \(b_P\) and \(b_N\), in terms of the incident terminal waves, \(a_P\) and \(a_N\):

\[
b_p = S_{11}a_p + S_{12}a_N \\
b_N = S_{21}a_P + S_{22}a_N
\]

Next, apply the definitions of Mixed Mode S-parameters and the differential and common mode power waves and note for symmetric, differential excitement: \(a_N=-a_P\), and for symmetric, common mode excitement, \(a_N=\pm a_P\):

\[
S_{DID1} = \frac{b_{DM}}{a_{DM}} = \frac{b_p - b_N}{a_p - a_N} = \frac{b_p - b_N}{\sqrt{2}} = \frac{S_{11}a_p + S_{12}a_N - S_{21}a_p - S_{22}a_N}{a_p - a_N} = \frac{S_{11}a_p - S_{12}a_p - S_{21}a_p + S_{22}a_p}{2a_p}
\]

\[
S_{DID1} = \frac{1}{2}(S_{11} - S_{12} - S_{21} + S_{22})
\]

This is the reflection coefficient associated with a reflected differential mode wave generated from a differential mode incident wave.

\[
S_{CIC1} = \frac{b_{CM}}{a_{CM}} = \frac{b_p + b_N}{a_p + a_N} = \frac{b_p + b_N}{\sqrt{2}} = \frac{S_{11}a_p + S_{12}a_N + S_{21}a_p + S_{22}a_N}{a_p + a_N} = \frac{S_{11}a_p + S_{12}a_p + S_{21}a_p + S_{22}a_p}{2a_p}
\]

\[
S_{CIC1} = \frac{1}{2}(S_{11} + S_{12} + S_{21} + S_{22})
\]
Similarly, this is reflection coefficient associated with a reflected common mode wave generated from common mode incident wave.

Two mixed mode reflection coefficients can be defined that cross-relate the differential-mode and common–mode.

\[ S_{\text{DIC1}} = \frac{b_{\text{DM}}}{a_{\text{CM}}} = \frac{b_p - b_N}{\sqrt{2}} = \frac{b_p - b_N}{a_p + a_N} = \frac{S_{11}a_p + S_{12}a_N - S_{21}a_p - S_{22}a_N}{a_p + a_N} = \frac{S_{11}a_p + S_{12}a_p - S_{21}a_p - S_{22}a_p}{2a_p} \]

\[ S_{\text{DIC1}} = \frac{1}{2}(S_{11} + S_{12} - S_{21} - S_{22}) \]

\[ S_{\text{CIDI}} = \frac{b_{\text{CM}}}{a_{\text{DM}}} = \frac{b_p + b_N}{\sqrt{2}} = \frac{b_p + b_N}{a_p - a_N} = \frac{S_{11}a_p + S_{12}a_N + S_{21}a_p + S_{22}a_N}{a_p - a_N} = \frac{S_{11}a_p - S_{12}a_p + S_{21}a_p - S_{22}a_p}{2a_p} \]

\[ S_{\text{CIDI}} = \frac{1}{2}(S_{11} - S_{12} + S_{21} - S_{22}) \]

In summary, the mixed mode S-parameters of the one port network, given a single-ended two-port representation can be expressed as follows:

\[ S_{\text{DIDI}} = \frac{1}{2}(S_{11} - S_{12} - S_{21} + S_{22}) \]

\[ S_{\text{CIDC1}} = \frac{1}{2}(S_{11} + S_{12} + S_{21} + S_{22}) \]

\[ S_{\text{DIC1}} = \frac{1}{2}(S_{11} + S_{12} - S_{21} - S_{22}) \]

\[ S_{\text{CIDI}} = \frac{1}{2}(S_{11} - S_{12} + S_{21} - S_{22}) \]

In the next article, the definitions will be used along with single-end two-port data to solve a mixed-mode impedance matching problem encountered in RF design.